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Phase transition from bubble state to stripe state for two-dimensional electrons in high Landau levels

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Abstract

Based on the Hartree–Fock approximation, we perform a detailed calculation of the cohesive energies of the round bubble phase, stripe phase, and possible intermediate states which may appear in the partially-filled high Landau levels in a two-dimensional electron system. Our computation shows that there is a first-order phase transition from bubbles to stripes at the filling factor of the uppermost Landau level $\nu^* \sim 0.35$. However, the almost complete independence of the cohesive energy from the bubble's shape near the phase transition point indicates that the real ground state may be very fragile before the critical filling factor.

The two-dimensional electron systems in high Landau levels (LLs) have aroused great interest, especially since the astonishing experiments of Lilly *et al* [1] and Du *et al* [2]. In contrast to the conventional quantum Hall effect observed in the $N = 0$ and 1 Landau levels, the experiments show that the magneto-transport property becomes highly anisotropic and nonlinear when more LLs ($N \geqslant 2$) are nearly half-filled ($\nu^* = \nu - 2N = 1/2$). The anisotropy of this system was recognized as due to the formation of unidirectional charge density waves (CDWs) or stripes. As a matter of fact, this stripe phase was predicted as the ground state when ν^* approaches $1/2$ in some previous theoretical studies [3,4]. Several years ago, Aleiner and Glazman [5] derived an effective interaction between the high LL electrons. Based on this interaction, Fogler, Koulakov and Shklovskii (FKS) [3] performed a Hartree–Fock (HF) study and argued that the stripe state wins against the uniform quantum liquid state when the highest LL is nearly half-filled, and that these stripes will break into bubbles when ν^* is far from $1/2$. Moessner and Chalker [4] soon generalized their conclusion and discussed the accuracy of the HF approximation under different circumstances.

Since these pioneering theoretical studies, in particular, after the remarkable experimental findings, great progress has been made toward the understanding of the physics in this system. On the experimental side, more interesting phenomena have been observed and reported, in some of which the subtle role of the in-plane magnetic field has been revealed [6–9]. On the theoretical side, the relationship between the giant transport anisotropy and the formation

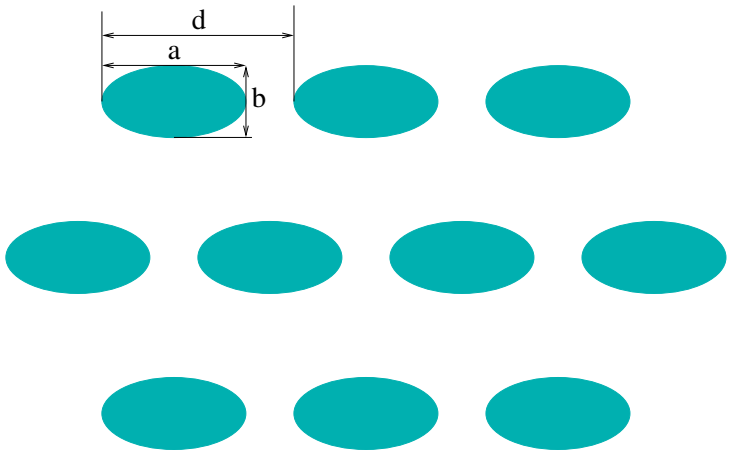

Figure 1. Equilateral triangular lattice with elliptical bubbles. The bubbles (shaded areas) are the regions where the electron guiding centres accumulate.

of stripes as well as the quantum fluctuations have been investigated in depth [10–16]. Additionally, the nonlinear effect can be qualitatively explained by the fact that the stripe is a quasi-one-dimensional object, and may be regarded as a coupled Luttinger liquid [10,17]. The effect of a tilted magnetic field on the anisotropic property has been discussed and compared with experimental results [9,18–20]. A possible mechanism for the re-entrant integral quantum Hall effect has been suggested [21].

In this paper we plan to calculate the cohesive energy of the various electron configurations, clarify the dependence of the cohesive energy on the bubble's shape and thus ascertain the order of the bubble-to-stripe phase transition. Our motivation comes from the following facts: on the one hand, the transition order is still a matter under dispute. The early study of FKS [3] asserted a first-order transition between these two phases, while some subsequent studies [10,11,22–24] implied that it may be a continuous one; up to now there seems to be no definite conclusion. On the other hand, the shape of the bubble and its effect on the cohesive energy has never received enough attention. However, this knowledge will provide the key for us to understand the type of the phase transition. Specifically, e.g., in a quasi-classical picture, the round bubbles distributed on a triangular lattice have the lowest cohesive energy when ν^* is small. As ν^* increases, if the elliptical bubbles take the place of the round ones in the ground state at some value of ν^*, they become steadily elongated throughout, and finally connect up into stripes when ν^* is close to $1/2$, then this can be regarded as a continuous phase transition. In contrast, if the imaginary elongated bubbles never take over as a ground state, or appear sporadically but are not stable so that the change is still discontinuous, then it can be asserted that a first-order phase transition has occured.

Based on the above analysis, we first construct a simple yet practical model to describe the bubble phase. As shown in figure 1, identical elliptical bubbles with long-axis a and short-axis b are embedded in the indefinite equilateral triangular lattice background, and the guiding centres of the electrons in the uppermost LL are uniformly distributed in and only in the bubbles. Here we use R and d to represent the ratio of a to b and the lattice constant, respectively. It can be easily seen that these parameters satisfy a simple relationship: $ab = \frac{2\sqrt{3}}{\pi}\nu^* d^2$.

Using this model as our starting-point, we can obtain a manifest picture of the electron configuration of the ground states and the corresponding cohesive energies at the various filling

factors ν^*. Specifically speaking, we slowly increase ν^* from a small value, and for each ν^*, we optimize the cohesive energy against the adjustable parameters R and d, i.e., the shape of the bubble and the lattice space, respectively, thus the particular configuration which has the lowest cohesive energy for a certain R and d will stand out as our ground state. The ratio R can range from 1 ($a = b$) to the maximal value $\pi/2\sqrt{3}\nu^*$ ($a = d$), the two limits representing the round bubble phase and the stripe phase, respectively. Our HF calculation based on this model rigidly shows that a first-order transition occurs between the bubble phase and stripe phase when $\nu^* \approx 0.35$.

In the following part we shall put forward our principal physical consideration and mathematical formulations. According to Aleiner and Glazman [5], we can ignore the LL mixing and consider the electrons in the fully filled lower LLs as an inert background sea. Consequently, we can focus on just the highest LL electrons with an effective interaction somewhat different from the Coulomb potential due to the screening of the lower LL electrons. The screening effect can be accounted for by the dielectric function

$$\varepsilon(q) = 1 + v(q)\Pi(q) \tag{1}$$

with

$$\Pi(q) = \frac{2}{\pi l^2} \sum_{i<N\leqslant j} \frac{(-1)^{j-i}}{\hbar\omega_c(j-i)} F_{ji}(q)F_{ij}(q) \tag{2}$$

and

$$F_{ji}(q) = L_i^{j-i}\left(\frac{q^2l^2}{2}\right)\mathrm{e}^{-\frac{q^2l^2}{4}} \tag{3}$$

where ω_c is the cyclotron frequency, $l = \frac{\hbar c}{eB}$ is the magnetic length, $v(q) = 2\pi e^2/\epsilon_0 q$ is the Fourier component of the Coulomb interaction with the bare dielectric constant and $L_i^j(x)$ is the associated Laguerre polynomial.

For the two-dimensional electrons confined in some definite LL, there is a simple but effective quasi-classical picture in which the electrons can be thought of as classical particles rotating around their own guiding centres with the frequency ω_c. The motion of the electrons can thus be reduced to that of the guiding centres. As FKS [3] have shown, because of the screening of the lower LLs, the interaction between the two electrons in the uppermost LL varies alternately between repulsive and attractive as their distance increases. The HF calculation shows that under such an effective interaction, the uniform distribution of the guiding centres is unstable for the formation of a CDW, or more precisely, the density wave of the guiding centres. For the CDW state, the cohesive energy per electron in the HF approximation is given by

$$E_{\mathrm{coh}}^{\mathrm{rec}} = \frac{n_L}{2\nu^*} \sum_{\vec{q}\neq 0} u_{HF}(q)|\Delta(\boldsymbol{q})|^2 \tag{4}$$

where $\Delta(\boldsymbol{q})$ is the CDW order parameter, $n_L = 1/2\pi l^2$ is the electron density, and the Hartree–Fock potential is defined in the same way as that in [25]

$$\begin{aligned} u_{HF}(q) &= u_H(q) - u_{ex}(q) \\ u_H(q) &= \frac{v(q)}{\varepsilon(q)} F_{NN}^2(q) \\ u_{ex}(q) &= 2\pi l^2 \int \frac{\mathrm{d}^2\boldsymbol{p}}{(2\pi)^2} \mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{q}l^2} u_H(p). \end{aligned} \tag{5}$$

Here the order parameter $\Delta(\boldsymbol{q})$ is the Fourier transformation of the guiding centre density $\nu(\boldsymbol{r})$ of the uppermost LL which equals *one* in the bubble and *zero* out of the bubble. It is

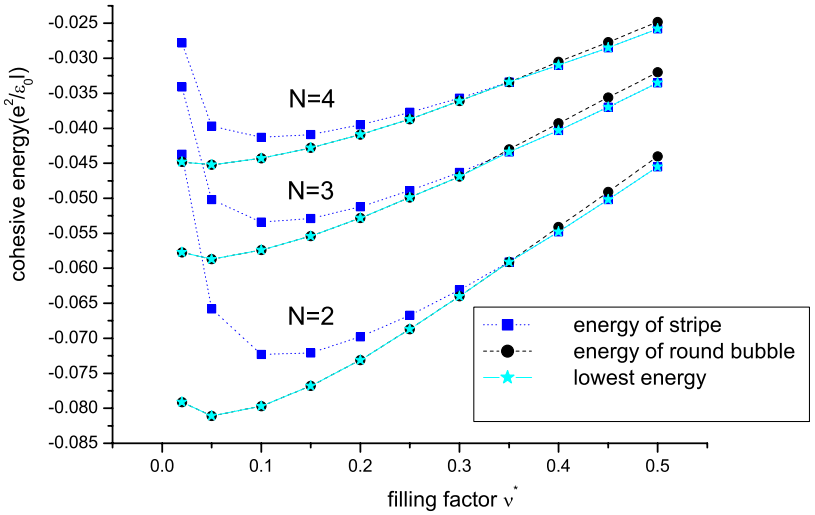

Figure 2. Cohesive energy versus filling factor in the highest Landau levels ($N = 2, 3, 4$). Here the unit of energy is $e^2/\epsilon_0 l$. Solid line with star symbols: the lowest cohesive energy; dashed line with round symbols: energy of the round bubbles on the triangular lattice; dotted line with square symbols: the stripe phase.

clear that $\Delta(\boldsymbol{q})$ is determined by the configuration parameters R and d. As mentioned above, for a given ν^*, we can find a $\Delta(\boldsymbol{q})$, i.e., R and d that minimizes the cohesive energy under a certain self-consistent condition imposed on $\Delta(\boldsymbol{q})$. In figure 2 we plot three curves for each LL of $N = 2, 3$ and 4 representing, respectively, the lowest cohesive energy, the energy of the round bubble state and that of the stripe state. The amount of the energy depends on the interaction parameter r_s [5]; here we choose the practically important case $r_s = 1.09$. It is easily seen that there is a transition point ν_c^*, before which the round symbols connected by a dashed line coincide with the star symbols connected by a solid line, indicating that the round bubble state has the lowest cohesive energy. Beyond ν_c^*, the square symbols connected by a dotted line coincide with the stars, indicating that the stripe state wins. The ν_c^* is about 0.35 for all the three LLs. This shows that the round bubble phase experiences a first-order transition and enters directly into the stripe phase.

To have a direct impression of the phase transition process as well as its nature, we present figures 3 and 4. In figure 3 we compare the cohesive energies of the round bubble state, stripe state and elliptical bubble state ($R = 1.7$), the last of which is a typical stripe crystal state in our terminology. It is shown that the energy of this imaginary elliptical state is either higher than that of the round bubble state before ν_c^*, or higher than that of the stripe state beyond ν_c^*, or higher than both near this point. The elliptical bubble states with other aspect ratios have the same property as stated above. Thus, the supposed intermediate states have never gained superiority simultaneously over both the round bubble state and the stripe state, so they will not appear as ground states as ν_c^* increases. From the above analysis we can conclude that, at the HF level, the bubble-to-stripe transition is a first-order one.

In figure 4 we depict the cohesive energies of the various elliptical bubble states with different aspect ratios R at some typical filling factors for $N = 2$, including the round bubble phase and stripe phase at the two ends of the R axis. We should note that the maximum value of R when $a = d$ is different for different filling factors. This figure reveals the nature of the phase transition in a straightforward way, and shows that the structure of the phase

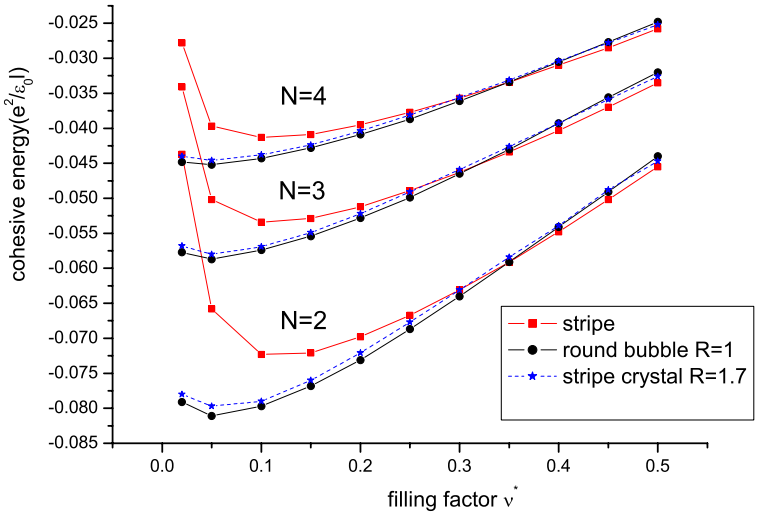

Figure 3. Cohesive energies of the three phases, bubble state, stripe crystal, and stripe state versus the filling factor ν^*.

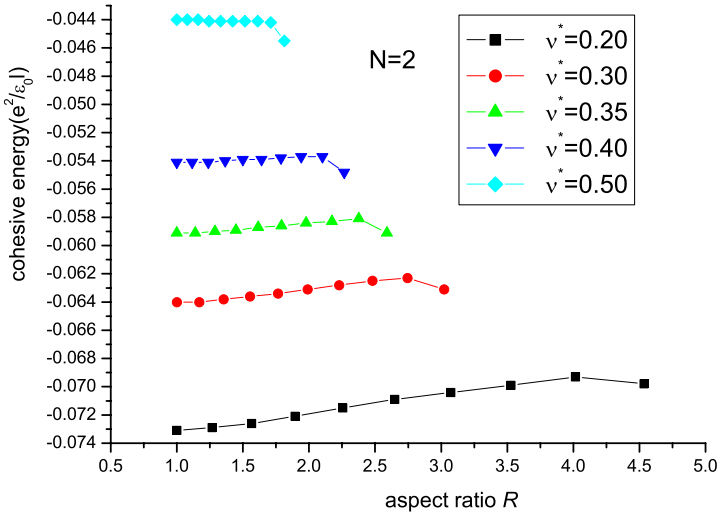

Figure 4. Cohesive energy of the elliptical bubbles versus the bubble's aspect ratio R for the high LL $N = 2$. We provide five series of points for $\nu^* = 0.20, 0.30, 0.35, 0.40$ and 0.50, respectively. The first and last points in each series correspond to the energies of the round bubble state and stripe state, respectively.

transition in our case is very similar to that of the classical first-order one. However, beyond the HF approximation our calculation can not rule out the possibility that the elliptical bubble state may appear occasionally due to quantum fluctuations and disorders, because the energies of the various bubbles with different aspect ratios are so close to the phase transition point. Quantitatively, in the $N = 2$ Landau level the maximal difference of the cohesive energies among the various bubbles is about 0.004 at $\nu^* = 1/5$ and 0.001 at $1/3$, in units of $e^2/\epsilon_0 l$. By comparison the difference between the cohesive energies of the CDW state and the Laughlin

liquid state in $N = 0$ Landau level is 0.006 at $\nu^* = 1/5$ and 0.021 at $1/3$ [21]. From this we can see that the difference among the energies of various states near the phase transition point $\nu^* = 1/3$ in the $N = 2$ Landau level is much more trivial than that between the CDW and Laughlin states in the lowest LL. Consequently, the true ground state near the phase transition point in high LLs is much more complicated.

Finally, we would like to make some comments. Enlightened by the complete independence of the cohesive energy on the bubble's shape just before the phase transition, we further argue that in this situation the two-dimensional electron system may not be in a strict 'bubble' phase but in an electron liquid crystal phase, that is, the electron bubbles may have no definite shape, size and regular arrangement under the strong quantum fluctuations. This phase has been predicted and studied in [10]. Again, we point out that the quantum fluctuations as well as the disorders may play an important role in the underlying physics. These factors may make it much more difficult to distinguish between states with almost similar energies.

In our view, therefore, the complete picture may be as follows: the highly-ordered 'bubble' phase first temporarily enters a relatively disordered liquid crystal phase just before the phase transition point, and then becomes a stable stripe phase. The above argument is only a conjecture, to verify which needs a careful and strict treatment of the quantum fluctuation, but that is not the purpose of this paper. We emphasize that this possible stripe crystal phase is not obvious in our HF calculation, but our results do imply that there may be very complicated phenomena just before the phase transition occurs.

In conclusion, we have computed the energies and configurations of the ground states based on the HF approximation, and have found a first-order transition from the bubble phase to the stripe phase. In addition, our results indicate that the system studied may have complicated behaviour near the phase transition point. During the preparation of this paper, we learned that Shibata and Yoshioka used the density matrix renormalization group method [26] to show that the bubble-to-stripe transition is of first-order, which strengthens our conclusion.

Acknowledgments

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